Mathematical Machine Learning Theory, M1407.002700
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Homework 5

These problems are part of homework 5. More problems will be assigned.

Problem 1: *Quadratic objectives in standard form II.* Let

$$F(\theta) = \frac{1}{2}\theta^\intercal H\theta + b^\intercal\theta + c,$$

where $H = H^\intercal \in \mathbb{R}^{d\times d}$ is positive semidefinite, $b \in \mathbb{R}^d$, and $c \in \mathbb{R}$.

(a) Assume $b \in \mathcal{R}(H)$. Show that there exists some $\theta^\star \in \mathbb{R}^d$ and $c' \in \mathbb{R}$ such that

$$F(\theta) = \frac{1}{2}(\theta - \theta^\star)^\intercal H(\theta - \theta^\star) + c'.$$

(b) Assume $b \notin \mathcal{R}(H)$. Show that $\inf_{\theta\in\mathbb{R}^d} F(\theta) = -\infty$.